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Roll waves study on Acquabona watershed: an application of mathematical modelling

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INTRODUCTION

Natural hazards are a subject each day more current and that has been broadly disclosed on news and media. As results from global warming, landscape usage and bad urban management, the climate change emerges as worldwide events, thus leading to natural catastrophes all across the globe (Burton, 2005). Those catastrophes are result of the impact of natural phenomena over vulnerable environments. In fact, they are in fact the conjunction of meteorological, geological/geomorphological effects with anthropic actions.

Also affected by climate changes, Brazil has confronted successive natural events such as floods and landslides during the rainy season (Pinho Francisco & Salgado, 2013). On Brazilian South and Southeast regions, mudflows are especially prominent during the rainy season over Serra do Mar, the mountains chains that extends along the shore (Guidicini & Nieble, 1984). According to Brazilian Ministry of Science and Technology, landslides and mudflows are the greatest cause of death among all Brazilian natural hazards.

Apart from the great damage incurred during Rio de Janeiro's event, the scenario established in the event (topography, mud constitution, etc.) was favorable to roll waves instabilities generation. However, properties such as front velocity propagation, mudflow/landslide depth from events of such nature haven't been recorded by Brazilian authorities and/or researchers which add up difficulty to any further investigation/conclusion. To analyze such kind of events based on mathematical models, the work here presented will explore an analogous debris/mud flow event recorded by Berti et al (2000) on Acquabona watershed, Cortina d'Ampezzo, Italy. The aim here is to identify how good a simplified mathematical model is on predicting roll waves formation on a natural environment as the presence of such phenomenon could be translated as a different level of risk to the environment.

From a mechanical point of view, the representation of mass-movement can be done through many physical approaches (Iverson, 1997). To choose a specific model over another relies on the representativeness of that model to the target phenomenon. Among them, mudflows and mud floods are natural events well represent by hydraulic approaches. They are known by their ability to erode soil and entrain sediment, sometimes even changing its rheological behaviour. From this last perspective, fluid in mudflows/mud floods can be Newtonian (clean water, low concentration of sediment, less than 1% in volume) or non-Newtonian (hyperconcentrated, high concentration of sediment, greater than 8% in volume). The

importance of the correct approach to risk management is clear when prediction of values such as flow height, front velocity, flow runout are at stake. Such task has been well developed, but still lack of application.

Besides all those aspects, secondary effects like bank erosion, front propagation and roll waves instabilities could make natural hazard events even more dangerous as they can change flow features and consequently interact more with the environment. The last, roll waves, are periodic free surface instabilities with considerable energy level (i.e. high amplitude and propagation velocity) which might appear on open channel flow configuration such as those we perceive on mountain regions and cliffs. Natural landscape can constitute slopes and preferable path for flows. Slope and fluid rheology could then reach (or surpass) threshold values for roll waves generation. In the past 10 years, several mathematical models have been developed on mud and debris flows, adding up complex features such as arbitrary topography, non-Newtonian fluid rheology and even assuming unstable conditions of the free surface (Van Asch, 2007; Balmforth & Mandre, 2004; Zanuttigh & Lamberti, 2007).

Roll waves appearance on artificial channel sections can lead to periodic overflows what induce debris entrainment and banks destabilization. From this perspective, they could not only elevate probability of mudflows events but also increase social and economic risks inherent to those hazards. Although no substantial proof of roll waves appearance have been taken on Brazil's territory, countries like New Zealand, China, Switzerland and Italy have already confirmed presence of surges and wave trains on mud flows over mountain regions (Zanuttigh & Lamberti, 2007).

Especially on Italy, an event at Cortina d'Ampezzo, on August 17th, 1998, draws attention regarding wave train detection, formed by almost periodic surges of mud and debris flow. This event, reported by Berti et al (2000), will be explored on this paper to promote a realistic data confrontation and a validation attempt to mathematical model described by (Maciel, Ferreira & Fiorot, 2013), which will be recalled on this work. Identify formation conditions and determine how roll waves instabilities will develop are task to be performed to better understand the risks dependence on such instabilities.

It's important to remark that the correct representation of fluids in this kind of phenomena requires attention. The variability found on solid-liquid fraction that composes the mixture contributes to the complexity of the fluid which could acquire cohesive or non-cohesive behaviour, depending on the material in suspension. Thus, exactly quantify shear ratio suffered from fluid due to stress applied is not usually possible. Mixtures where sediment concentration is high will be characterized mainly by the presence of a yield stress and viscous interactions, usually called mud flows. On the other hand, when dispersive turbulent stress are remarkably noticed and become the main mechanism of energy dispersion characterize mud floods and debris flow events.

The variability of physical properties found *in situ* during or even after events (mud floods, mud flows, debris flow) gives most of the difficulty that arises when categorizing those events. Conversely, even though mathematical approximations and modelling are usually employed, often they don't properly represent realistic scenarios. Usually data employed on those models come from experimental data and/or *in situ* measurements sometimes poorly collected, i.e. their representation of wholeness of event, environment, etc. are not complete. This paper will show the applicability of a 1-D model, based on shallow water equations and viscoplastic rheology, to that kind of event. No turbulent dispersion should be considered. Roll waves properties estimative (amplitude, wavelength, propagation velocity) will be made. The event used in this paper was reported by Berti et al (2000) and make reference to a

debris flow event from 1998 in Acquabona watershed, Cortina d'Ampezzo, Italy. However, authors did not have quantified the contribution of turbulent dispersions to the stress tensor.

MATHEMATICAL AND NUMERICAL MODELING OF ROLL WAVES

The modeling departs from a mechanical point of view, considering a constant discharge rate (q [m²/s]) of mud and debris aqueous mixture with volumetric concentration ϕ_v , composing an incompressible viscous fluid with density ρ [kg/m³] flowing down an inclined open channel (θ degrees), where the driven force is gravity, represented by its acceleration (g [m²/s]). For the compounded mixture (water + clay or water + fine sand + clay) investigated in this work, rheometric studies from Coussot (1994), Piau (1996), Huang & Garcia (1998), Maciel, Santos & Ferreira (2009) and many others proved that when in absence of sedimentation effects, fluid rheology can be described by the Herschel-Bulkley non-linear rheological model with permanent simple shearing conditions, as shown by the following expression:

$$\tau_{xz} = \tau_c + K_n \left(\frac{\partial u}{\partial z} \right)^n, \text{ for } \tau_{xz} > \tau_c, \quad (1)$$

elsewhere $\partial u / \partial z = 0$. On Eq. 1, τ_{xz} is the shear stress magnitude acting on longitudinal coordinate x due to a gradient in vertical coordinate z ; u is the longitudinal velocity, $\partial u / \partial z$ is the shear rate magnitude. This model is then inserted into the viscous part of the stress tensor on Cauchy's equation.

The flow is then considered laminar, where the Reynolds number $Re = \rho(u_0^2 - n h_0^n) / K_n$ should be not greater than 500, and in shallow water conditions, where mathematical approximations lead to a Saint-Venant alike system. Equations are then vertically averaged and mean values for flow height (h_0 [m]) and velocity (u_0 [m/s]) are found. Through these values, a dimensionless system is constituted where three main dimensionless numbers are identified: the Froude number $Fr = u_0 / \sqrt{gh_0 \cos \theta}$; the dimensionless yield stress $C^* = \tau_c / (\rho g h_0 \sin \theta)$; and the flow index n . The roll wave equation is given in a mobile coordinate system ($x' = x^* - U^* t^*$), as follows:

$$\frac{\partial h^*}{\partial x'} = \frac{h^* - C^* - (1 - C^*) \left[(1 + U^*(h^* - 1)) \left(\frac{1 - C^*}{h^* - C^*} \right) \left(\frac{n + 1 + nC^*}{(n + 1)h^* + nC^*} \right) \right]^n}{(\alpha - 1)U^{*2} - \frac{\alpha(1 - U^*)^2}{h^{*2}} + \frac{h^*}{Fr^2}}. \quad (2)$$

where U is roll wave celerity, computed as:

$$U^* = \frac{U}{u_0} = \alpha + \sqrt{\alpha^2 - \alpha + Fr^{-2}}, \quad (3)$$

And α is a momentum correction factor that assumes values between 1 and 1.2 (for complete plug flow and pure laminar, respectively). This equation should have continuous solution inside the wavelength if sufficient conditions are reached. The domain of possible solution depends on flow Froude number, which should be greater than the threshold Fr_{min} , found through linear stability analysis (Trowbridge, 1987; Coussot, 1994; Maciel, Ferreira & Fiorot, 2013) and given by the Equation (4):

$$Fr > Fr_{min} = \frac{\vartheta}{\sqrt{\varphi^2 - 2\alpha\varphi\vartheta + \alpha\vartheta^2}}, \quad (4)$$

where $\varphi(n, C^*) = (n + 1)(2n + 1) / (n + 1 + nC^*)$ and $\vartheta(n, C^*) = n(1 - C^*)$.

Wavelength λ is calculated through defined integration of Equation (2) inverted, between wave trough and crest (h_1 and h_2 , respectively). Then, $h^*(x')$ is given by defined integration of Equation (2) inside wavelength λ . Finally, dimensionless results are brought to scale through mean flow parameters in steady and permanent configuration. All steps and details of this mathematical model can be found at Maciel, Ferreira & Fiorot (2013). Schematically, roll waves properties can be described as shows Figure 1:

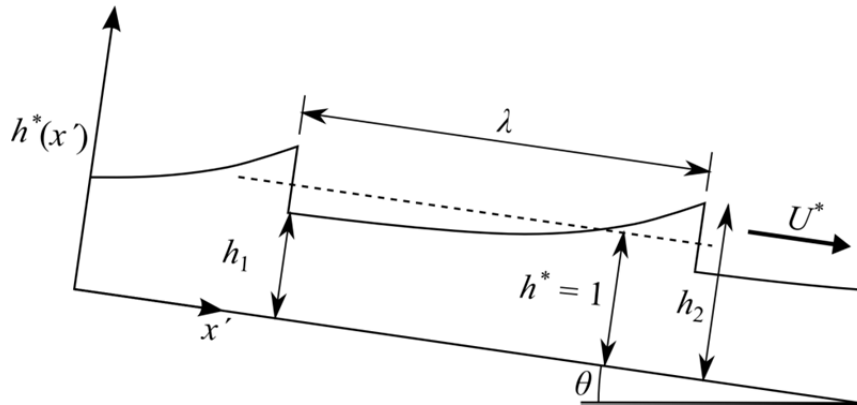


Figure 1. Dimensionless schematics on roll waves properties for permanent flow. Main physical properties are remarked. Mean flow height h_0 is a singularity point of roll waves equation.

Finally, the model utilization can be summarized as shows Figure 2. This methodology will be adopted in this article to estimate roll waves formation on a realistic case scenario, based on the event detailed by Berti et al (2000). The model here applied was only calibrated to fluid materials having silt, clay and sand in its matrix, corresponding mostly to the granulometry variability of downstream material related during the chosen Acquabona event.

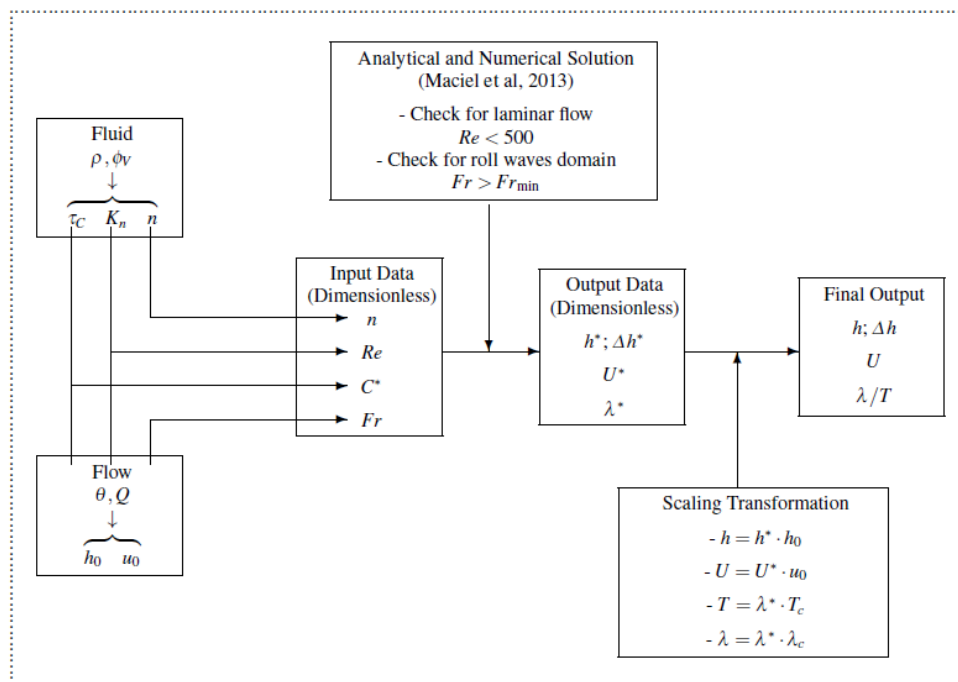


Figure 2. Diagram showing input and output data of roll waves prediction methodology. T_c and λ_c are characteristic time and length used for scaling process, following Maciel, Ferreira & Fiorot (2013).

MODELING ACQUABONA EVENT

The purpose of this work is to explore *in situ* measurements collected by Berti et al (2000) and find out if roll waves were prone to develop under those circumstances and what would have been their main features. Based on the mathematical model recalled before, this work will estimate roll waves properties that could have happened on Acquabona site based on basic information given by those authors such as precipitation, landscape, sediment material displaced, etc.

Site description

The Acquabona channel is localized on the Dolomites Mountains, Northeast Italy, Southeast from Cortina d'Ampezzo city, having downstream the Boite river valley. Morphopedologic properties of region are prone to mass movement natural events that occur almost every year during rainy season and for this reason is explored by researchers for studies. The upstream region is composed by a limestone rock basin, presenting 30° steepness. Along the channel, the superficial soil is composed by rock and gravel from Dolomites and present mean inclination of 18°. Downstream region reaches Boite river, crossing road Roma, and presents 7° steepness. The total length of the channel is 1632 meters.

The fluid formed from the mass movement changes as it flows down the channel. The viscous matrix is the result of an aqueous solution of gravel (with mean diameter between 2 and 20 mm) and sand (diameter less than 2 mm). The flow is capable of dislodge rocks and boulders up to 2 meter diameter. Fluid fraction of rock with diameter greater than 20 mm can reach 28% of total mixture mass.

During flow evolution from upstream to downstream, a densification process is remarked by fluid samples taken from past events (Berti et al; 1999; Berti et al, 2000) which can be explained by soil composition of intermediate part of the channel that is more susceptible to be eroded. Upstream, roughly 60% of fluid matrix is gravel while rough sands (from diameter between 65 µm and 2 mm) and fines (silt and clay with diameter smaller than 65 µm) have 35% and 5% of its composition. At intermediate length, fines quantities are greater, reaching 20% in volume of fluid matrix. Downstream, fines reach 30% of mixture, pointing out channel erodibility.

Measurements stations are installed at strategic position along the channel, allowing data acquisition from events. Data collected comprise pluviosity, wind speed, surges velocity (geophones), flow height (ultrasonic devices), flow superficial velocity (image processing) and pressure gauges. Manipulation of these data will make possible estimative of fluid rheological properties and flow dynamics (discharge rate) which along site topography constitute all necessary data input for model.

Event sample

During summer of 1998, at August 17th, a large debris flow event occurred on Acquabona channel when data were collected by 3 measurement stations installed along the channel. Stations were positioned at specific points of the channel to facilitate comprehension of event evolution. The first station, installed 300 meters from channel inlet (rock basin), allowed observations from cameras (image data, later processed into surface velocity information), pluviosity, surge velocity and pressure from the event. The second station, positioned on the intermediate part of the channel, just wind speed and surge velocity data were collected. The last station, installed 500 meters away from channel outlet close to Boite river, image data, flow height surges velocities were collected. All data, methods and descriptions are related

on Berti et al work (Berti et al, 2000). Measurements from pluviosity done at inlet station showed that the rain that initiated the event reached 25 mm during 30 minutes and that the first debris surge happened after 45 minutes of rain, when pluviosity reached 20 mm. The total volume displaced estimated was 8000-9000 m³.

Startup mechanisms of bed mobilization were elucidated and discussed by the authors in details. However, the observation concerning the event of August 17th, 1998, was particularly interesting as more than 15 surges of mud and debris were identified during 35 minutes after the first surge, as shows figure 1. Nevertheless, the stage where the wave trains is detected is highly unpredictable and waves with non-constant amplitude, celerity and wavelength are measured. This could be explained as we deal with a natural phenomenon where a broad set of frequencies could have disturbed the system, thus altering waves on its transitional stage. Another important remark concerns the equipment used for measurements which has great sensibility depending on the roughness of the surface (ultrasound system *versus* shock wave). We should then identify a well-behaved set of waves which could be a good sample of a roll wave and gather from it all input parameters necessary for its numerical simulation and prediction.

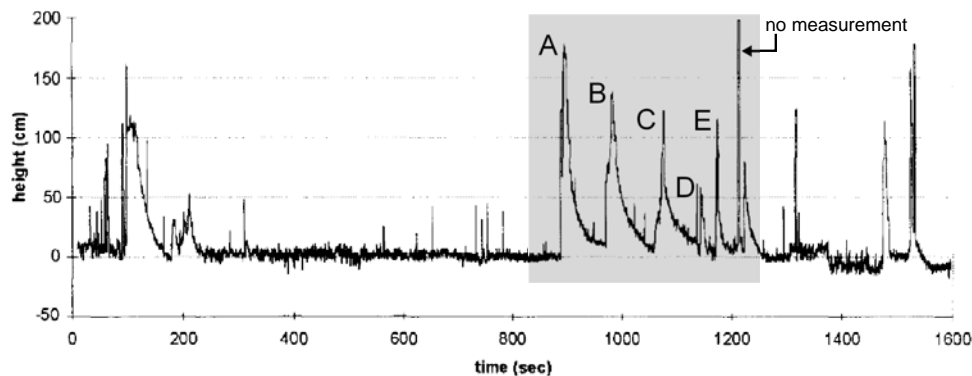


Figure 3. Measurements from station 3, downstream channel, collected through ultrasonic equipment on August 17th, 1998, related by Berti et al (2000). Highlighted sequence of surges identifies possible roll wave formation used in this work.

The fronts identified by A, B, C, D and E on Figure 3 present relatively well-behaved amplitude and wavelength when compared others event fronts. As known, ultrasonic devices suffer from great losses of signal when sharp/steep properties are at stake. Then, considering this system flaw, measurements could have suffered from an intense scattering when larger boulder passed through, thus showing high peaks followed by low valleys as shows Figure 3 in the end of highlighted area where no measurement are registered. Therefore, we will identify the end of our wave train on 1209 seconds, where this elevated peak abruptly appears on data series. Characteristics of surges are then collected directly from data and showed on Table 1.

Table 1. Characteristics of identified surges. Values directly collected from Berti et al (2000) measurements.

| Surge | Time [s] | U [m/s] | Δh [m] | T [s] |
|------------|----------|-----------|----------------|---------|
| A | 895 | 6.67 | 1.58 | 87 |
| B | 982 | 5.80 | 1.28 | 93 |
| C | 1075 | 7.69 | 1.07 | 67 |
| D | 1142 | 5.88 | 0.50 | 33 |
| E | 1175 | 5.00 | 1.06 | 34 |
| Mean value | | 6.21 | 1.10 | 63 |

Parameters gathering

First of all, results collected from pressure gauges showed that fluid pressure was equal to the bottom shear stress, thus leading to the conclusion that flow was liquefied. This observation allows employment of rheology models for mixture representation and obtaining of others flow characteristics as will follow.

Secondly, qualitative observations and data collected from soil show that sediment material density (from rocks, gravel and sands) should be around 2000 kg/m^3 . Concerning mixture composition, according to Zanuttigh & Lamberti (2007), the concentration was roughly estimated by 60% of sediment in water expressed in terms of mass. Then, from mass concentration, we can compute volume concentration which should be around 43%, and the mixture density on 1430 kg/m^3 . As explained by Ancey (2007), mixtures mainly composed by clay (colloidal mixtures) have strong non-Newtonian behavior for which the yield stress exponentially increases with material concentration. However, when sand takes part into the mixture, at high shear rate, inert particles of sand take part into the flow, configuring sediments inertial regimen Bagnold (1954), thus changing rheology behavior of mixture. Experimental analysis shows that when concentration of colloidal material is kept and concentration of inert material is added up, non-Newtonian effects are reduced. In fact, this last mixture represents how the fluid should have behaved on the case-scenario of Acquabona. Based on experimental rheological studies (Coussot, 1994; Ancey, 2007; Maciel, Santos & Ferreira, 2009), fluid yield stress (τ_c) is roughly estimated on 200 Pa. Concerning index flow, results from the same author show that it stabilizes around 0.30, which is not far from 1/3 calibrated for French torrential lava (Coussot, 1994). Although difficulty to predict rheological parameters, consistency index K_n can be estimated and should assume values between 50 and 1000 Pas^n based on the same works.

And finally, the two main flow parameters have to be found: mean flow velocity u_0 and mean flow height h_0 . In order to guarantee model validity, we should assure that flow is at laminar regime, i.e. Reynolds number not greater than 500. As a value of reference, a first estimative of flow regime can be considered through the maximum flow velocity, which is the surface velocity u_s . Values obtained after the first surge and before the wave train ($t < 900 \text{ s}$) suggests that mean surface velocity was about 3.4 m/s. As roll waves crests and troughs originated around mean flow height (Ng & Mei, 1994; Pascal, 1999), one way to estimate h_0 is to calculate the mean value between all waves' crests and troughs. Based on the register from the station, mean flow height should be seen around 0.65 meters. However, the scoured depth of bed should also be considered for further calculations as it doesn't appear directly on the register. Thus, adding the scoured depth (0.25 meters) to the mean value calculated before (0.65 meters), $h_0 \approx 0.90$ meters. Flow Reynolds number should not be greater than 160 (for $K_n = 50 \text{ Pas}^n$). Flow main parameters greatly influence model response to input parameters, so another method is used for sake of comparison.

Since we dispose from measured waves celerity U , we developed an iterative process to compute h_0 through Equation (3) and u_0 , through surface velocity u_s as relates Equation (5):

$$u_0 = \left(\frac{n + 1 + nC^*}{2n + 1} \right) u_s. \quad (5)$$

Assuming that the momentum coefficient distribution is equal to 1 in the beginning and that we have a mean value for waves propagation velocity equal to 6.21 m/s (mean value calculated from Table 1), variables are solved and rapidly converge to constant values for $h_0 = 0.91$ meters and $u_0 = 2.85$ m/s. Values show good representation of flow properties, in

agreement with first assumptions. We consider that the precision is greater to front propagation velocity estimative and then the last value for h_0 and u_0 are used as input parameter in the model. Fluid and mean properties of flow are summarized on Table 2 as well as dimensionless input parameters to 1D model.

Table 2. Fluid and flow mean properties and input parameters for 1D model simulation.

| Fluid properties | | Flow properties | | Dimensionless parameters | |
|------------------|--------------------------|-----------------|----------|--------------------------|-------|
| n | 0.3 | h_0 | 0.91 m | Fr_{min} (Eq. 4) | 0.21 |
| τ_c | 200 Pa | u_0 | 2.85 m/s | Fr | 0.96 |
| K_n | 50-1000 Pas ⁿ | θ | 7° | C^* | 0.13 |
| ρ | 1430 kg/m ³ | | | Re | 8-160 |

Prediction results

Based on the input parameters set from Table 2, roll waves estimate is performed and results are shown on Figure 4. All waves represented here are the greatest roll waves possible, i.e. the ones with greater amplitude and longest period, given a set of input parameters. This assumption is possible because when flow disturbance is composed by many frequencies, the lowest disturbance frequency will be responsible for permanent waves generated (Kranenburg, 1992). Sensitivity analysis is also performed supposing that we had an uncertainty of 10% in the wave celerity, which would then change h_0 and u_0 , following the reasoning explained before. All waves characteristics are summarized in Table 3, along with measured waves characteristics from Berti et al (2000) for sake of comparison.

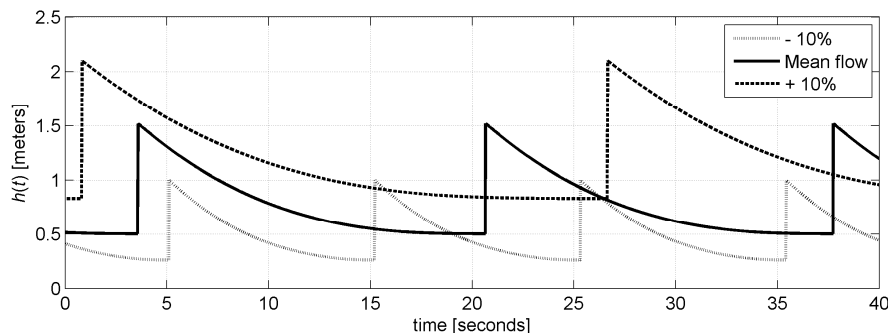


Figure 4. Waves' profiles obtained for Acquabona event from August 17th, 1998. Wave expected for mean flow configuration is simulated and represented by dark line. Two other cases are showed for $\pm 10\%$ variation on mean waves celerity estimative.

Table 3. Output data from mathematical/numerical simulation from 1D model to respect of Acquabona event from August 17th, 1998 in comparison with direct results from event measurements.

| | Measured Berti et al (2000) | Model prediction | | |
|------------|--------------------------------|------------------|------|-------|
| | | - 10% | Mean | + 10% |
| U | 6.21 | 5.59 | 6.22 | 6.84 |
| Δh | 1.10 | 0.74 | 1.02 | 1.27 |
| T | 63 | 10 | 17 | 25 |

Not only flow characteristics are important but also rheological properties of fluid play an important role. Based on typical results given by the model from mean values from measures, we can evaluate how waves' properties would change as rheological parameters vary. The assumption that sediment concentration would change along the channel as bed scouring happened is completely plausible. Two other tests cases are simulated considering that fluid could assume a less non-Newtonian behavior ($n = 0.5$; $\tau_c = 50$ Pa) and a more

non-Newtonian behavior ($n = 0.2$; $\tau_c = 500$ Pa). These results are presented in comparison with the reference test case for mean flow properties on Figure 5 are summarized in Table 4.

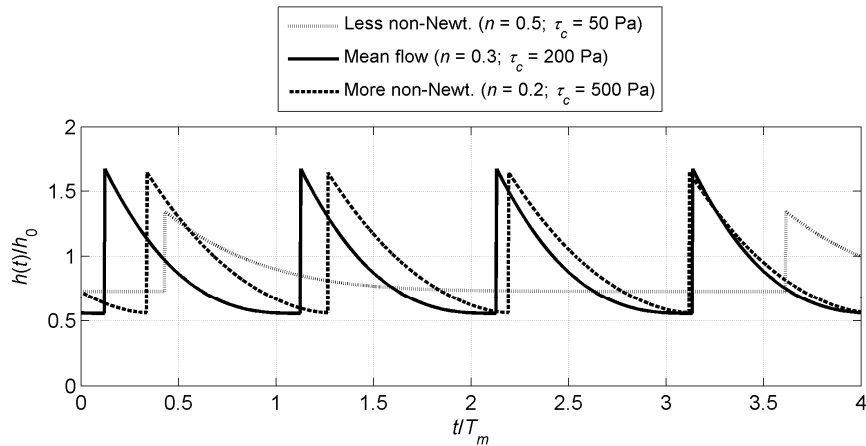


Figure 5. Influence of rheological properties (n and τ_c) on waves profiles $h(t)$. Profiles $h(t)$ are showed as function of time t , scaled to the each mean flow height and to period of waves on mean flow prediction T_m , respectively.

Table 4. Output data from mathematical/numerical simulation from 1D model to respect of Acquabona event from August 17th, 1998 in comparison with direct results from event measurements, when assuming rheological properties variations.

| | Measured Berti et al (2000) | Model prediction | | |
|------------|--------------------------------|--|------|---|
| | | Less non-Newtonian $n = 0.5$; $\tau_c = 50$ Pa | Mean | More non-Newtonian $n = 0.2$; $\tau_c = 500$ Pa |
| U | 6.21 | 6.83 | 6.22 | 6.83 |
| Δh | 1.10 | 0.89 | 1.02 | 1.40 |
| T | 63 | 54 | 17 | 16 |

FINAL REMARKS

The first evidence is that the simulation made from the mean value for h_0 have good agreement to the mean amplitude measured, although showing weak correspondence in wave period. The poor agreement between period/wavelength calculated and measured could be a systematic flaw of the model the mathematical model which neglects effects such as surface tension, fluid densification, variations in fluids properties, and others. However, it's also important to note that as we suppose variations on measured properties (thus slightly changing model input parameters) a considerable influence on model results is perceived, reaching higher wave periods and amplitudes. Simulated waves properties can significantly vary depending on rheological properties of fluid. As it can be seen, when non-Newtonian effects are greater, wave amplitude increase and period decreases, although the proportionality to respect of its mean flow does not change significantly. Conversely, when rheological effects are smaller, period is greater, and get closer to measured values, but on the contrary, amplitude decrease.

From an Engineering perspective, results show that the model is capable to estimate maximum roll wave in mudflows events. The results presented show that model is capable to reasonably identify the most important features (amplitude, period and propagation velocity) of possible roll wave formation in mudflow events. However, both flow and rheological properties of fluid significantly affect results on wave properties. This points out that the correct and detailed measurement of properties play an important role to estimative of possible events.

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